Math 679V **Topics in Topology: Stable Commutator Length** Spring 2014

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For a group G, the commutator subgroup $[G, G] \subseteq G$ consists of those elements that are expressible as a product of simple commutators $[g, h] = ghg^{-1}h^{-1}$. An element $g \in [G, G]$ typically can be expressed as product of commutators in several ways, the commutator length, $\mathrm{cl}(g)$, is least number of commutators needed in any such expression. It is often quite difficult to compute $\mathrm{cl}(g)$ for a given element, but there is a stabilization that has several useful interpretations and applications. The *stable commutator length* of an element $g \in [G, G]$ is:

$$\mathrm{scl}(g) = \lim_{n\to\infty} \frac{\mathrm{cl}(g^n)}{n}$$

There are two alternative descriptions of scl: as a measure of complexity of a surface bounded by g and as a (pseudo-)norm on $B_1(G)$, the space of 1–boundaries in G. As such stable commutator length uses ideas from algebra, functional analysis and topology. These alternative characterizations allow us to compute scl in some cases.

Specific topics to be covered include:

- algebraic and topological definitions of scl
- Calegari's Rationality Theorem
- analytic interpretation of scl, Bavard Duality
- geometric estimates of scl using quasi-morphisms

Prerequisite: Foundations of Topology (Math 5703) or instructor's consent