

1. The Laplace transform of the differential equation yields

$$\mathcal{L}\{y\} = \frac{1}{s-3}e^{-2s}$$

so that

$$y = e^{3(t-2)}\mathcal{U}(t-2).$$

3. The Laplace transform of the differential equation yields

$$\mathcal{L}\{y\} = \frac{1}{s^2+1}(1+e^{-2\pi s})$$

so that

$$y = \sin t + \sin t\mathcal{U}(t-2\pi).$$

7. The Laplace transform of the differential equation yields

$$\mathcal{L}\{y\} = \frac{1}{s^2+2s}(1+e^{-s}) = \left[\frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2}\right](1+e^{-s})$$

so that

$$y = \frac{1}{2} - \frac{1}{2}e^{-2t} + \left[\frac{1}{2} - \frac{1}{2}e^{-2(t-1)}\right]\mathcal{U}(t-1).$$

11. The Laplace transform of the differential equation yields

$$\begin{aligned}\mathcal{L}\{y\} &= \frac{4+s}{s^2+4s+13} + \frac{e^{-\pi s} + e^{-3\pi s}}{s^2+4s+13} \\ &= \frac{2}{3}\frac{3}{(s+2)^2+3^2} + \frac{s+2}{(s+2)^2+3^2} + \frac{1}{3}\frac{3}{(s+2)^2+3^2}(e^{-\pi s} + e^{-3\pi s})\end{aligned}$$

so that

$$\begin{aligned}y &= \frac{2}{3}e^{-2t}\sin 3t + e^{-2t}\cos 3t + \frac{1}{3}e^{-2(t-\pi)}\sin 3(t-\pi)\mathcal{U}(t-\pi) \\ &\quad + \frac{1}{3}e^{-2(t-3\pi)}\sin 3(t-3\pi)\mathcal{U}(t-3\pi).\end{aligned}$$