**19.** $\mathscr{L}\left\{1 * t^3\right\} = \dfrac{1}{s}\dfrac{3!}{s^4} = \dfrac{6}{s^5}$

**23.** $\mathscr{L}\left\{\displaystyle\int_0^t e^{\tau}\, d\tau\right\} = \dfrac{1}{s}\mathscr{L}\{e^t\} = \dfrac{1}{s(s-1)}$

**37.** The Laplace transform of the given equation is

$$\mathscr{L}\{f\} + \mathscr{L}\{t\}\mathscr{L}\{f\} = \mathscr{L}\{t\}.$$

Solving for $\mathscr{L}\{f\}$ we obtain $\mathscr{L}\{f\} = \dfrac{1}{s^2+1}$. Thus, $f(t) = \sin t$.

**49.** $\mathscr{L}\{f(t)\} = \dfrac{1}{1-e^{-2as}}\left[\displaystyle\int_0^a e^{-st}dt - \int_a^{2a} e^{-st}dt\right] = \dfrac{(1-e^{-as})^2}{s(1-e^{-2as})} = \dfrac{1-e^{-as}}{s(1+e^{-as})}$

**51.** Using integration by parts,

$$\mathscr{L}\{f(t)\} = \frac{1}{1-e^{-bs}}\int_0^b \frac{a}{b}te^{-st}dt = \frac{a}{s}\left(\frac{1}{bs} - \frac{1}{e^{bs}-1}\right).$$