1. $\mathscr{L}\left\{te^{10t}\right\} = \dfrac{1}{(s-10)^2}$

3. $\mathscr{L}\left\{t^3e^{-2t}\right\} = \dfrac{3!}{(s+2)^4}$

7. $\mathscr{L}\left\{e^t \sin 3t\right\} = \dfrac{3}{(s-1)^2+9}$

11. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s+2)^3}\right\} = \mathscr{L}^{-1}\left\{\dfrac{1}{2}\,\dfrac{2}{(s+2)^3}\right\} = \dfrac{1}{2}t^2e^{-2t}$

13. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2-6s+10}\right\} = \mathscr{L}^{-1}\left\{\dfrac{1}{(s-3)^2+1^2}\right\} = e^{3t}\sin t$

15. $\mathscr{L}^{-1}\left\{\dfrac{s}{s^2+4s+5}\right\} = \mathscr{L}^{-1}\left\{\dfrac{s+2}{(s+2)^2+1^2} - 2\dfrac{1}{(s+2)^2+1^2}\right\} = e^{-2t}\cos t - 2e^{-2t}\sin t$

21. The Laplace transform of the differential equation is

$$s\,\mathscr{L}\{y\} - y(0) + 4\,\mathscr{L}\{y\} = \frac{1}{s+4}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{1}{(s+4)^2} + \frac{2}{s+4}.$$

Thus

$$y = te^{-4t} + 2e^{-4t}.$$

23. The Laplace transform of the differential equation is

$$s^2\,\mathscr{L}\{y\} - sy(0) - y'(0) + 2[s\,\mathscr{L}\{y\} - y(0)] + \mathscr{L}\{y\} = 0.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{s+3}{(s+1)^2} = \frac{1}{s+1} + \frac{2}{(s+1)^2}.$$

Thus

$$y = e^{-t} + 2te^{-t}.$$

25. The Laplace transform of the differential equation is

$$s^2\,\mathscr{L}\{y\} - sy(0) - y'(0) - 6\,[s\,\mathscr{L}\{y\} - y(0)] + 9\,\mathscr{L}\{y\} = \frac{1}{s^2}\,.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{1+s^2}{s^2(s-3)^2} = \frac{2}{27}\,\frac{1}{s} + \frac{1}{9}\,\frac{1}{s^2} - \frac{2}{27}\,\frac{1}{s-3} + \frac{10}{9}\,\frac{1}{(s-3)^2}\,.$$

Thus

$$y = \frac{2}{27} + \frac{1}{9}t - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}.$$