

$$1. \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{1}{2} \mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = \frac{1}{2}t^2$$

$$3. \mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{24} \cdot \frac{4!}{s^5}\right\} = t - 2t^4$$

$$5. \mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + 3 \cdot \frac{1}{s^2} + \frac{3}{2} \cdot \frac{2}{s^3} + \frac{1}{6} \cdot \frac{3!}{s^4}\right\} = 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$$

$$7. \mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\} = t - 1 + e^{2t}$$

$$9. \mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4} \mathcal{L}^{-1}\left\{\frac{1}{s+1/4}\right\} = \frac{1}{4}e^{-t/4}$$

$$11. \mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{7} \cdot \frac{7}{s^2+49}\right\} = \frac{5}{7} \sin 7t$$

$$13. \mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1/4}\right\} = \cos \frac{1}{2}t$$

31. The Laplace transform of the initial-value problem is

$$s \mathcal{L}\{y\} - y(0) - \mathcal{L}\{y\} = \frac{1}{s}.$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = -\frac{1}{s} + \frac{1}{s-1}.$$

Thus

$$y = -1 + e^t.$$

33. The Laplace transform of the initial-value problem is

$$s \mathcal{L}\{y\} - y(0) + 6 \mathcal{L}\{y\} = \frac{1}{s-4}.$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{1}{(s-4)(s+6)} + \frac{2}{s+6} = \frac{1}{10} \cdot \frac{1}{s-4} + \frac{19}{10} \cdot \frac{1}{s+6}.$$

Thus

$$y = \frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}.$$

35. The Laplace transform of the initial-value problem is

$$s^2 \mathcal{L}\{y\} - sy(0) - y'(0) + 5[s \mathcal{L}\{y\} - y(0)] + 4 \mathcal{L}\{y\} = 0.$$

Solving for $\mathcal{L}\{y\}$ we obtain

$$\mathcal{L}\{y\} = \frac{s+5}{s^2+5s+4} = \frac{4}{3} \frac{1}{s+1} - \frac{1}{3} \frac{1}{s+4}.$$

Thus

$$y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}.$$