

29. If $\frac{1}{2}x'' + \frac{1}{2}x' + 6x = 10 \cos 3t$, $x(0) = 2$, and $x'(0) = 0$ then

$$x_c = e^{-t/2} \left(c_1 \cos \frac{\sqrt{47}}{2} t + c_2 \sin \frac{\sqrt{47}}{2} t \right)$$

and $x_p = \frac{10}{3}(\cos 3t + \sin 3t)$ so that the equation of motion is

$$x = e^{-t/2} \left(-\frac{4}{3} \cos \frac{\sqrt{47}}{2} t - \frac{64}{3\sqrt{47}} \sin \frac{\sqrt{47}}{2} t \right) + \frac{10}{3}(\cos 3t + \sin 3t).$$

33. From $2x'' + 32x = 68e^{-2t} \cos 4t$, $x(0) = 0$, and $x'(0) = 0$ we obtain $x_c = c_1 \cos 4t + c_2 \sin 4t$ and $x_p = \frac{1}{2}e^{-2t} \cos 4t - 2e^{-2t} \sin 4t$ so that

$$x = -\frac{1}{2} \cos 4t + \frac{9}{4} \sin 4t + \frac{1}{2}e^{-2t} \cos 4t - 2e^{-2t} \sin 4t.$$

37. From $x'' + 4x = -5 \sin 2t + 3 \cos 2t$, $x(0) = -1$, and $x'(0) = 1$ we obtain $x_c = c_1 \cos 2t + c_2 \sin 2t$, $x_p = \frac{3}{4}t \sin 2t + \frac{5}{4}t \cos 2t$, and

$$x = -\cos 2t - \frac{1}{8} \sin 2t + \frac{3}{4}t \sin 2t + \frac{5}{4}t \cos 2t.$$

45. Solving $\frac{1}{20}q'' + 2q' + 100q = 0$ we obtain $q(t) = e^{-20t}(c_1 \cos 40t + c_2 \sin 40t)$. The initial conditions $q(0) = 5$ and $q'(0) = 0$ imply $c_1 = 5$ and $c_2 = 5/2$. Thus

$$q(t) = e^{-20t} \left(5 \cos 40t + \frac{5}{2} \sin 40t \right) = \sqrt{25 + 25/4} e^{-20t} \sin(40t + 1.1071)$$

and $q(0.01) \approx 4.5676$ coulombs. The charge is zero for the first time when $40t + 1.1071 = \pi$ or $t \approx 0.0509$ second.

47. Solving $\frac{5}{3}q'' + 10q' + 30q = 300$ we obtain $q(t) = e^{-3t}(c_1 \cos 3t + c_2 \sin 3t) + 10$. The initial conditions $q(0) = q'(0) = 0$ imply $c_1 = c_2 = -10$. Thus

$$q(t) = 10 - 10e^{-3t}(\cos 3t + \sin 3t) \quad \text{and} \quad i(t) = 60e^{-3t} \sin 3t.$$

Solving $i(t) = 0$ we see that the maximum charge occurs when $t = \pi/3$ and $q(\pi/3) \approx 10.432$.