

1. From $Dx = 2x - y$ and $Dy = x$ we obtain $y = 2x - Dx$, $Dy = 2Dx - D^2x$, and $(D^2 - 2D + 1)x = 0$.
The solution is

$$x = c_1 e^t + c_2 t e^t$$

$$y = (c_1 - c_2) e^t + c_2 t e^t.$$

5. From $(D^2 + 5)x - 2y = 0$ and $-2x + (D^2 + 2)y = 0$ we obtain $y = \frac{1}{2}(D^2 + 5)x$, $D^2y = \frac{1}{2}(D^4 + 5D^2)x$, and $(D^2 + 1)(D^2 + 6)x = 0$. The solution is

$$x = c_1 \cos t + c_2 \sin t + c_3 \cos \sqrt{6}t + c_4 \sin \sqrt{6}t$$

$$y = 2c_1 \cos t + 2c_2 \sin t - \frac{1}{2}c_3 \cos \sqrt{6}t - \frac{1}{2}c_4 \sin \sqrt{6}t.$$

9. From $Dx + D^2y = e^{3t}$ and $(D + 1)x + (D - 1)y = 4e^{3t}$ we obtain $D(D^2 + 1)x = 34e^{3t}$ and $D(D^2 + 1)y = -8e^{3t}$. The solution is

$$y = c_1 + c_2 \sin t + c_3 \cos t - \frac{4}{15}e^{3t}$$

$$x = c_4 + c_5 \sin t + c_6 \cos t + \frac{17}{15}e^{3t}.$$

Substituting into $(D + 1)x + (D - 1)y = 4e^{3t}$ gives

$$(c_4 - c_1) + (c_5 - c_6 - c_3 - c_2) \sin t + (c_6 + c_5 + c_2 - c_3) \cos t = 0$$

so that $c_4 = c_1$, $c_5 = c_3$, $c_6 = -c_2$, and

$$x = c_1 - c_2 \cos t + c_3 \sin t + \frac{17}{15}e^{3t}.$$

13. From $(2D - 5)x + Dy = e^t$ and $(D - 1)x + Dy = 5e^t$ we obtain $Dy = (5 - 2D)x + e^t$ and $(4 - D)x = 4e^t$.
Then

$$x = c_1 e^{4t} + \frac{4}{3}e^t$$

and $Dy = -3c_1 e^{4t} + 5e^t$ so that

$$y = -\frac{3}{4}c_1 e^{4t} + c_2 + 5e^t.$$

17. From $Dx = y$, $Dy = z$, and $Dz = x$ we obtain $x = D^2y = D^3x$ so that $(D - 1)(D^2 + D + 1)x = 0$,

$$x = c_1 e^t + e^{-t/2} \left[c_2 \sin \frac{\sqrt{3}}{2}t + c_3 \cos \frac{\sqrt{3}}{2}t \right],$$

$$y = c_1 e^t + \left(-\frac{1}{2}c_2 - \frac{\sqrt{3}}{2}c_3 \right) e^{-t/2} \sin \frac{\sqrt{3}}{2}t + \left(\frac{\sqrt{3}}{2}c_2 - \frac{1}{2}c_3 \right) e^{-t/2} \cos \frac{\sqrt{3}}{2}t,$$

and

$$z = c_1 e^t + \left(-\frac{1}{2}c_2 + \frac{\sqrt{3}}{2}c_3 \right) e^{-t/2} \sin \frac{\sqrt{3}}{2}t + \left(-\frac{\sqrt{3}}{2}c_2 - \frac{1}{2}c_3 \right) e^{-t/2} \cos \frac{\sqrt{3}}{2}t.$$