

1. The auxiliary equation is $m^2 - m - 2 = (m + 1)(m - 2) = 0$ so that $y = c_1x^{-1} + c_2x^2$.
3. The auxiliary equation is $m^2 = 0$ so that $y = c_1 + c_2 \ln x$.
5. The auxiliary equation is $m^2 + 4 = 0$ so that $y = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)$.
7. The auxiliary equation is $m^2 - 4m - 2 = 0$ so that $y = c_1x^{2-\sqrt{6}} + c_2x^{2+\sqrt{6}}$.
9. The auxiliary equation is $25m^2 + 1 = 0$ so that $y = c_1 \cos\left(\frac{1}{5} \ln x\right) + c_2 \sin\left(\frac{1}{5} \ln x\right)$.
19. The auxiliary equation is $m^2 - 5m = m(m - 5) = 0$ so that $y_c = c_1 + c_2x^5$ and

$$W(1, x^5) = \begin{vmatrix} 1 & x^5 \\ 0 & 5x^4 \end{vmatrix} = 5x^4.$$

Identifying $f(x) = x^3$ we obtain $u_1' = -\frac{1}{5}x^4$ and $u_2' = 1/5x$. Then $u_1 = -\frac{1}{25}x^5$, $u_2 = \frac{1}{5} \ln x$, and

$$y = c_1 + c_2x^5 - \frac{1}{25}x^5 + \frac{1}{5}x^5 \ln x = c_1 + c_3x^5 + \frac{1}{5}x^5 \ln x.$$

31. Substituting $x = e^t$ into the differential equation we obtain

$$\frac{d^2y}{dt^2} + 8 \frac{dy}{dt} - 20y = 0.$$

The auxiliary equation is $m^2 + 8m - 20 = (m + 10)(m - 2) = 0$ so that

$$y = c_1e^{-10t} + c_2e^{2t} = c_1x^{-10} + c_2x^2.$$

33. Substituting $x = e^t$ into the differential equation we obtain

$$\frac{d^2y}{dt^2} + 9 \frac{dy}{dt} + 8y = e^{2t}.$$

The auxiliary equation is $m^2 + 9m + 8 = (m + 1)(m + 8) = 0$ so that $y_c = c_1e^{-t} + c_2e^{-8t}$. Using undetermined coefficients we try $y_p = Ae^{2t}$. This leads to $30Ae^{2t} = e^{2t}$, so that $A = 1/30$ and

$$y = c_1e^{-t} + c_2e^{-8t} + \frac{1}{30}e^{2t} = c_1x^{-1} + c_2x^{-8} + \frac{1}{30}x^2.$$