In Problems 1-8 we use reduction of order to find a second solution. In Problems 9-16 we use formula (5) from the text.

1. The auxiliary equation is $m^2 + 1 = 0$, so $y_c = c_1 \cos x + c_2 \sin x$ and

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1.$$

Identifying $f(x) = \sec x$ we obtain

$$u_1' = -\frac{\sin x \sec x}{1} = -\tan x$$

$$u_2' = \frac{\cos x \sec x}{1} = 1.$$

Then $u_1 = \ln|\cos x|$, $u_2 = x$, and

$$y = c_1 \cos x + c_2 \sin x + \cos x \ln|\cos x| + x \sin x.$$

In Problems 1-8 we use reduction of order to find a second solution. In Problems 9-16 we use formula (5) from the text.

3. The auxiliary equation is $m^2 + 1 = 0$, so $y_c = c_1 \cos x + c_2 \sin x$ and

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1.$$

Identifying $f(x) = \sin x$ we obtain

$$u_1' = -\sin^2 x$$

$$u_2' = \cos x \sin x.$$

Then

$$u_1 = \frac{1}{4}\sin 2x - \frac{1}{2}x = \frac{1}{2}\sin x \cos x - \frac{1}{2}x$$

$$u_2 = -\frac{1}{2}\cos^2 x.$$

and

$$y = c_1 \cos x + c_2 \sin x + \frac{1}{2}\sin x \cos^2 x - \frac{1}{2}x \cos x - \frac{1}{2}\cos^2 x \sin x$$

$$= c_1 \cos x + c_2 \sin x - \frac{1}{2}x \cos x.$$

In Problems 1-8 we use reduction of order to find a second solution. In Problems 9-16 we use formula (5) from the text.

5. The auxiliary equation is $m^2 + 1 = 0$, so $y_c = c_1 \cos x + c_2 \sin x$ and

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1.$$

Identifying $f(x) = \cos^2 x$ we obtain

$$u_1' = -\sin x \cos^2 x$$
$$u_2' = \cos^3 x = \cos x \left(1 - \sin^2 x\right).$$

Then $u_1 = \frac{1}{3}\cos^3 x$, $u_2 = \sin x - \frac{1}{3}\sin^3 x$, and

$$\begin{aligned} y &= c_1 \cos x + c_2 \sin x + \frac{1}{3}\cos^4 x + \sin^2 x - \frac{1}{3}\sin^4 x \\ &= c_1 \cos x + c_2 \sin x + \frac{1}{3}\left(\cos^2 x + \sin^2 x\right)\left(\cos^2 x - \sin^2 x\right) + \sin^2 x \\ &= c_1 \cos x + c_2 \sin x + \frac{1}{3}\cos^2 x + \frac{2}{3}\sin^2 x \\ &= c_1 \cos x + c_2 \sin x + \frac{1}{3} + \frac{1}{3}\sin^2 x. \end{aligned}$$

In Problems 1-8 we use reduction of order to find a second solution. In Problems 9-16 we use formula (5) from the text.

7. The auxiliary equation is $m^2 - 1 = 0$, so $y_c = c_1e^x + c_2e^{-x}$ and

$$W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -2.$$

Identifying $f(x) = \cosh x = \frac{1}{2}(e^{-x} + e^x)$ we obtain

$$u_1' = \frac{1}{4}e^{-2x} + \frac{1}{4}$$

$$u_2' = -\frac{1}{4} - \frac{1}{4}e^{2x}.$$

Then

$$u_1 = -\frac{1}{8}e^{-2x} + \frac{1}{4}x$$

$$u_2 = -\frac{1}{8}e^{2x} - \frac{1}{4}x$$

and

$$\begin{aligned} y &= c_1e^x + c_2e^{-x} - \frac{1}{8}e^{-x} + \frac{1}{4}xe^x - \frac{1}{8}e^x - \frac{1}{4}xe^{-x} \\ &= c_3e^x + c_4e^{-x} + \frac{1}{4}x(e^x - e^{-x}) \\ &= c_3e^x + c_4e^{-x} + \frac{1}{2}x\sinh x. \end{aligned}$$