**1.** $(9D^2 - 4)y = (3D - 2)(3D + 2)y = \sin x$

**3.** $(D^2 - 4D - 12)y = (D - 6)(D + 2)y = x - 6$

**11.** $D^4y = D^4(10x^3 - 2x) = D^3(30x^2 - 2) = D^2(60x) = D(60) = 0$

**15.** $D^4$ because of $x^3$

**29.** $e^{6x}$, $e^{-3x/2}$

**35.** Applying $D$ to the differential equation we obtain

$$D(D^2 - 9)y = 0.$$

Then

$$y = \underbrace{c_1e^{3x} + c_2e^{-3x}}_{y_c} + c_3$$

and $y_p = A$. Substituting $y_p$ into the differential equation yields $-9A = 54$ or $A = -6$. The general solution is

$$y = c_1e^{3x} + c_2e^{-3x} - 6.$$

**37.** Applying $D$ to the differential equation we obtain

$$D(D^2 + D)y = D^2(D + 1)y = 0.$$

Then

$$y = \underbrace{c_1 + c_2e^{-x}}_{y_c} + c_3x$$

and $y_p = Ax$. Substituting $y_p$ into the differential equation yields $A = 3$. The general solution is

$$y = c_1 + c_2e^{-3x} + 3x.$$

**39.** Applying $D^2$ to the differential equation we obtain

$$D^2(D^2+4D+4)y = D^2(D+2)^2y = 0.$$

Then

$$y = \underbrace{c_1e^{-2x} + c_2xe^{-2x}}_{y_c} + c_3 + c_4x$$

and $y_p = Ax + B$. Substituting $y_p$ into the differential equation yields $4Ax + (4A + 4B) = 2x + 6$. Equating coefficients gives

$$4A = 2$$

$$4A + 4B = 6.$$

Then $A = 1/2$, $B = 1$, and the general solution is

$$y = c_1e^{-2x} + c_2xe^{-2x} + \frac{1}{2}x + 1.$$

**41.** Applying $D^3$ to the differential equation we obtain

$$D^3(D^3 + D^2)y = D^5(D+1)y = 0.$$

Then

$$y = \underbrace{c_1 + c_2x + c_3e^{-x}}_{y_c} + c_4x^4 + c_5x^3 + c_6x^2$$

and $y_p = Ax^4 + Bx^3 + Cx^2$. Substituting $y_p$ into the differential equation yields

$$12Ax^2 + (24A + 6B)x + (6B + 2C) = 8x^2.$$

Equating coefficients gives

$$12A = 8$$

$$24A + 6B = 0$$

$$6B + 2C = 0.$$

Then $A = 2/3$, $B = -8/3$, $C = 8$, and the general solution is

$$y = c_1 + c_2x + c_3e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2.$$