1. From $y = c_1e^x + c_2e^{-x}$ we find $y' = c_1e^x - c_2e^{-x}$. Then $y(0) = c_1 + c_2 = 0$, $y'(0) = c_1 - c_2 = 1$ so that $c_1 = \frac{1}{2}$ and $c_2 = -\frac{1}{2}$. The solution is $y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$.

3. From $y = c_1x + c_2x\ln x$ we find $y' = c_1 + c_2(1 + \ln x)$. Then $y(1) = c_1 = 3$, $y'(1) = c_1 + c_2 = -1$ so that $c_1 = 3$ and $c_2 = -4$. The solution is $y = 3x - 4x\ln x$.

15. Since $(-4)x + (3)x^2 + (1)(4x - 3x^2) = 0$ the set of functions is linearly dependent.

17. Since $(-1/5)5 + (1)\cos^2 x + (1)\sin^2 x = 0$ the set of functions is linearly dependent.

19. Since $(-4)x + (3)(x - 1) + (1)(x + 3) = 0$ the set of functions is linearly dependent.