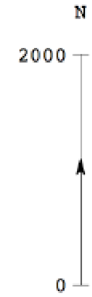
1. **(a)** Solving $N(1 - 0.0005N) = 0$ for N we find the equilibrium solutions $N = 0$ and $N = 2000$. When $0 < N < 2000$, $dN/dt > 0$. From the phase portrait we see that $\lim_{t\to\infty} N(t) = 2000$. A graph of the solution is shown in part (b).

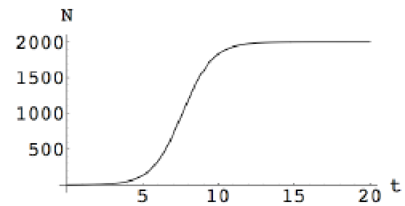


(b) Separating variables and integrating we have

$$\frac{dN}{N(1 - 0.0005N)} = \Big(\frac{1}{N} - \frac{1}{N - 2000}\Big)dN = dt$$

and

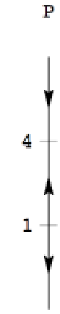
$$\ln N - \ln(N - 2000) = t + c.$$



Solving for N we get $N(t) = 2000e^{c+t}/(1 + e^{c+t}) = 2000e^c e^t/(1 + e^c e^t)$. Using $N(0) = 1$ and solving for e^c we find $e^c = 1/1999$ and so $N(t) = 2000e^t/(1999 + e^t)$. Then $N(10) = 1833.59$, so 1834 companies are expected to adopt the new technology when $t = 10$.

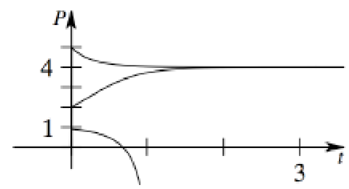
3. From $dP/dt = P\left(10^{-1} - 10^{-7}P\right)$ and $P(0) = 5000$ we obtain $P = 500/(0.0005 + 0.0995e^{-0.1t})$ so that $P \to 1{,}000{,}000$ as $t \to \infty$. If $P(t) = 500{,}000$ then $t = 52.9$ months.

5. **(a)** The differential equation is $dP/dt = P(5-P)-4$. Solving $P(5-P)-4=0$ for P we obtain equilibrium solutions $P=1$ and $P=4$. The phase portrait is shown on the right and solution curves are shown in part (b). We see that for $P_0>4$ and $1<P_0<4$ the population approaches 4 as t increases. For $0<P<1$ the population decreases to 0 in finite time.



(b) The differential equation is

$$\frac{dP}{dt} = P(5-P)-4 = -(P^2-5P+4) = -(P-4)(P-1).$$



Separating variables and integrating, we obtain

$$\frac{dP}{(P-4)(P-1)} = -dt$$

$$\left(\frac{1/3}{P-4} - \frac{1/3}{P-1}\right)dP = -dt$$

$$\frac{1}{3}\ln\left|\frac{P-4}{P-1}\right| = -t+c$$

$$\frac{P-4}{P-1} = c_1e^{-3t}.$$

Setting $t=0$ and $P=P_0$ we find $c_1=(P_0-4)/(P_0-1)$. Solving for P we obtain

$$P(t) = \frac{4(P_0-1)-(P_0-4)e^{-3t}}{(P_0-1)-(P_0-4)e^{-3t}}.$$

(c) To find when the population becomes extinct in the case $0<P_0<1$ we set $P=0$ in

$$\frac{P-4}{P-1} = \frac{P_0-4}{P_0-1}e^{-3t}$$

from part (a) and solve for t. This gives the time of extinction

$$t = -\frac{1}{3}\ln\frac{4(P_0-1)}{P_0-4}.$$

7. Solving $P(5-P)-7=0$ for P we obtain complex roots, so there are no equilibrium solutions. Since $dP/dt<0$ for all values of P, the population becomes extinct for any initial condition. Using separation of variables to solve the initial-value problem, we get

$$P(t)=\frac{5}{2}+\frac{\sqrt{3}}{2}\tan\left[\tan^{-1}\left(\frac{2P_0-5}{\sqrt{3}}\right)-\frac{\sqrt{3}}{2}t\right].$$

Solving $P(t)=0$ for t we see that the time of extinction is

$$t=\frac{2}{3}\left(\sqrt{3}\tan^{-1}(5/\sqrt{3})+\sqrt{3}\tan^{-1}[(2P_0-5)/\sqrt{3}]\right).$$

9. Let $X=X(t)$ be the amount of C at time t and $dX/dt=k(120-2X)(150-X)$. If $X(0)=0$ and $X(5)=10$, then

$$X(t)=\frac{150-150e^{180kt}}{1-2.5e^{180kt}},$$

where $k=.0001259$ and $X(20)=29.3$ grams. Now by L'Hôpital's rule, $X\to 60$ as $t\to\infty$, so that the amount of $A\to 0$ and the amount of $B\to 30$ as $t\to\infty$.