1. Letting $y = ux$ we have

$$\begin{aligned}(x - ux)\,dx + x(u\,dx + x\,du) &= 0\\ dx + x\,du &= 0\\ \frac{dx}{x} + du &= 0\\ \ln|x| + u &= c\\ x\ln|x| + y &= cx.\end{aligned}$$

3. Letting $x = vy$ we have

$$\begin{aligned}vy(v\,dy + y\,dv) + (y - 2vy)\,dy &= 0\\ vy^2\,dv + y\left(v^2 - 2v + 1\right)dy &= 0\\ \frac{v\,dv}{(v-1)^2} + \frac{dy}{y} &= 0\\ \ln|v - 1| - \frac{1}{v-1} + \ln|y| &= c\\ \ln\left|\frac{x}{y} - 1\right| - \frac{1}{x/y - 1} + \ln y &= c\\ (x - y)\ln|x - y| - y &= c(x - y).\end{aligned}$$

5. Letting $y = ux$ we have

$$\begin{aligned}\left(u^2x^2 + ux^2\right)dx - x^2(u\,dx + x\,du) &= 0\\ u^2\,dx - x\,du &= 0\\ \frac{dx}{x} - \frac{du}{u^2} &= 0\\ \ln|x| + \frac{1}{u} &= c\\ \ln|x| + \frac{x}{y} &= c\\ y\ln|x| + x &= cy.\end{aligned}$$

11. Letting $y = ux$ we have

$$\begin{aligned}\left(x^3 - u^3x^3\right)dx + u^2x^3(u\,dx + x\,du) &= 0\\ dx + u^2x\,du &= 0\\ \frac{dx}{x} + u^2\,du &= 0\\ \ln|x| + \frac{1}{3}u^3 &= c\\ 3x^3\ln|x| + y^3 &= c_1x^3.\end{aligned}$$

Using $y(1) = 2$ we find $c_1 = 8$. The solution of the initial-value problem is $3x^3\ln|x| + y^3 = 8x^3$.

17. From $y' + y = xy^4$ and $w = y^{-3}$ we obtain $\dfrac{dw}{dx} - 3w = -3x$. An integrating factor is e^{-3x} so that $e^{-3x}w = xe^{-3x} + \frac{1}{3}e^{-3x} + c$ or $y^{-3} = x + \frac{1}{3} + ce^{3x}$.

21. From $y' - \dfrac{2}{x}y = \dfrac{3}{x^2}y^4$ and $w = y^{-3}$ we obtain $\dfrac{dw}{dx} + \dfrac{6}{x}w = -\dfrac{9}{x^2}$. An integrating factor is x^6 so that $x^6w = -\frac{9}{5}x^5 + c$ or $y^{-3} = -\frac{9}{5}x^{-1} + cx^{-6}$. If $y(1) = \frac{1}{2}$ then $c = \frac{49}{5}$ and $y^{-3} = -\frac{9}{5}x^{-1} + \frac{49}{5}x^{-6}$.