

1. For  $y' - 5y = 0$  an integrating factor is  $e^{-\int 5 dx} = e^{-5x}$  so that  $\frac{d}{dx} [e^{-5x}y] = 0$  and  $y = ce^{5x}$  for  $-\infty < x < \infty$ . There is no transient term.
3. For  $y' + y = e^{3x}$  an integrating factor is  $e^{\int dx} = e^x$  so that  $\frac{d}{dx} [e^x y] = e^{4x}$  and  $y = \frac{1}{4}e^{3x} + ce^{-x}$  for  $-\infty < x < \infty$ . The transient term is  $ce^{-x}$ .
7. For  $y' + \frac{1}{x}y = \frac{1}{x^2}$  an integrating factor is  $e^{\int (1/x)dx} = x$  so that  $\frac{d}{dx} [xy] = \frac{1}{x}$  and  $y = \frac{1}{x} \ln x + \frac{c}{x}$  for  $0 < x < \infty$ . The entire solution is transient.
11. For  $y' + \frac{4}{x}y = x^2 - 1$  an integrating factor is  $e^{\int (4/x)dx} = x^4$  so that  $\frac{d}{dx} [x^4 y] = x^6 - x^4$  and  $y = \frac{1}{7}x^3 - \frac{1}{5}x + cx^{-4}$  for  $0 < x < \infty$ . The transient term is  $cx^{-4}$ .
25. For  $y' + \frac{1}{x}y = \frac{1}{x}e^x$  an integrating factor is  $e^{\int (1/x)dx} = x$  so that  $\frac{d}{dx} [xy] = e^x$  and  $y = \frac{1}{x}e^x + \frac{c}{x}$  for  $0 < x < \infty$ . If  $y(1) = 2$  then  $c = 2 - e$  and  $y = \frac{1}{x}e^x + \frac{2 - e}{x}$ .
27. For  $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$  an integrating factor is  $e^{\int (R/L)dt} = e^{Rt/L}$  so that  $\frac{d}{dt} [e^{Rt/L} i] = \frac{E}{L}e^{Rt/L}$  and  $i = \frac{E}{R} + ce^{-Rt/L}$  for  $-\infty < t < \infty$ . If  $i(0) = i_0$  then  $c = i_0 - E/R$  and  $i = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right)e^{-Rt/L}$ .