Review Sheet

Math 2584C

Welcome back! This class builds on top of your previous work in Calculus, so you need to make sure you are ready to work with that material. In fact most of the techniques of differential equations reduce the problem to a question of algebra or calculus. This sheet should help you identify areas you need to work on.

Things you may never do. Some common errors that we do not enjoy seeing:

- You cannot expand a fraction by "distributing" a sum in the denominator: $\frac{1}{a+b}$ is very different from $\frac{1}{a} + \frac{1}{b}$. Similarly, you cannot "distribute" over denominators even if you manipulate the numerators: $\frac{c+d}{a+b}$ is different from $\frac{c}{a} + \frac{d}{b}$.

- You cannot expand a binomial to a power by "distributing" the exponent: $(a + b)^r$ is not the same as $a^r + b^r$.

- You cannot manipulate an integral by "distributing" over a product of functions:

$$\int f(x)g(x)dx \neq \left(\int f(x)dx \right)\left(\int g(x)dx \right).$$

Things to recall from algebra. You must know how to:

- Manipulate fractions correctly.

- Factor quadratic polynomials and certain higher order polynomials.

- Apply logarithm rules.

Things to recall from trigonometry. You must know:

- The definitions of sin, cos, sec, tan, cot and csc.

- Special values of sin and cos.

- Basic trigonometric identities, including Pythagorean and sum-angle identities.

Things to recall from calculus. You must be able to:

- Differentiate power functions, trig functions, inverse trig functions, exponentials and logarithms.

- Differentiate complicated expressions using linearity, the product rule, the quotient rule, and if necessary, logarithms.

- Recognize and perform elementary integrals, including power functions, basic trig functions and their derivatives, and exponential functions.

- Perform substitution (u-substitution) to evaluate integrals.

- Perform integration by parts.

- Use trigonometric substitutions to evaluate integrals.

- Use partial fractions to evaluate integrals.

- Recognize appropriate techniques of integration to apply to a given integral.

- Evaluate tricky integrals that require using more than one technique of integration.

- Write down power series expressions for familiar functions.

- Manipulate power series.

- Evaluate convergence of series.

Review problems.

1. Find all real solutions to $x^4 + 4x^2 - 5 = 0$.

2. Find all real solutions for x in $x^2 + Ax + B = 0$ in terms of the parameters A and B.

3. Differentiate $f(x) = \tan(x) + \sec(x)$.

4. Differentiate $f(x) = \arctan(x)\arcsin(x)$.

5. Differentiate $f(x) = x^x$.

6. Differentiate
$$f(x) = \frac{\sqrt{x}}{e^x}.$$

7. Differentiate
$$f(x) = \frac{\sqrt{x^3}}{\sqrt[5]{x^4}}.$$

8. Write down a power series expression for e^{2x}.

9. Find the interval of convergence of $\sum_{k=0}^{\infty} 2x^k$.

Evaluate the integrals:

10.
$$\int xe^{x^2}\,dx$$

11.
$$\int \frac{dt}{t + \sqrt{t}}$$

12.
$$\int xe^x\,dx$$

13.
$$\int \sin(y)e^y\,dy$$

14.
$$\int \sec^3(s)\,ds$$

15.
$$\int \arctan(x)\,dx$$

16.
$$\int \frac{3\sin(t) + 2\sin(t)\cos(t)}{3 - \sin^2(t) + 3\cos(t)}\,dt$$

17.
$$\int \frac{1}{\sqrt{1 - x^2}^3}\,dx$$

18.
$$\int t^2 \ln(t)\,dt$$

19.
$$\int \frac{v^2 + v + 1}{v^3 + v}\,dv$$