MATH 5703	**Topology I**	Fall 2014
Section: 001	MWF 10:45 – 11:35	Prof. Matthew Clay
	MAIN 322	

Homework 9

For these exercises, let M be a connected 1–manifold, $U, V \subseteq M$ connected open sets and $\phi\colon U \to \mathbb{R}$, $\psi\colon V \to \mathbb{R}$ local charts, i.e., embeddings.

The *boundary* of a subset A of a topological space is $\partial A = \overline{A} - \mathrm{Int}(A)$.

For hints and more details, see: `http://www.jstor.org/stable/2322421`.

Gale, David. *The Teaching of Mathematics: The Classification of 1–Manifolds: A Take-Home Exam.* Amer. Math. Monthly 94 (1987), no. 2, 170–175.

1. Suppose that both $U \cap V$ and $U - V$ are nonempty and let (x_n) be a sequence in $U \cap V$ that converges to a point $x \in U - V$. Show that the set $\{\psi(x_n)\}$ does not have a limit point in $\psi(V)$.

2. Suppose that $U \cap V$, $U - V$ and $V - U$ are nonempty and let W be a connected component of $U \cap V$. Show that $\partial\phi(W) \cap \partial\phi(U) \neq \emptyset$ and $\partial\psi(W) \cap \partial\psi(V) \neq \emptyset$. Conclude that $U \cap V$ has at most two connected components.

3. Show that if $U \cap V$ has two components, then M is homeomorphic to S^1.

4. Suppose that $U \cap V$, $U - V$ and $V - U$ are nonempty and $U \cap V$ is connected. Show that there is an embedding $\rho\colon U \cup V \to \mathbb{R}$.

5. Prove that M is homeomorphic to either $\mathbb{R}$ or S^1.