MATH 5703	**Topology I**	Fall 2014
Section: 001	MWF 10:45 – 11:35	Prof. Matthew Clay
	MAIN 322	

Homework 8

1. Show that a continuous bijection between compact Hausdorff spaces is a homeomorphism.

2. Let $\{A_n, f_n\}$ and $\{B_n, g_n\}$ be inverse limit systems and suppose that for each n there is a continuous function $\varphi_n \colon A_n \to B_n$ such that $g_n \varphi_n = \varphi_{n-1} f_n$. In other words, the following diagram commutes:

$$\begin{array}{ccc} A_n & \xrightarrow{f_n} & A_{n-1} \\ {\scriptstyle \varphi_n}\downarrow & & \downarrow{\scriptstyle \varphi_{n-1}} \\ B_n & \xrightarrow{g_n} & B_{n-1} \end{array}$$

Show that there is an induced continuous function $\varphi \colon \varprojlim A_n \to \varprojlim B_n$ defined by $\varphi((a_n)) = (\varphi_n(a_n))$

3. Let $\{X_n, f_n\}$ be a inverse limit system where each X_n is a compact Hausdorff space. Prove that the inverse limit space $\varprojlim X_n$ is a compact Hausdorff space.

4. Let $\mathcal{C}$ be the Cantor set. Prove that $\mathcal{C}$ is homeomorphic to $\mathcal{C} \times \mathcal{C}$.

5. Let $\mathcal{C}$ be the Cantor set. Show that every continuous function $f \colon \mathcal{C} \to \mathbb{R}^n$ can be extended to a continuous function $F \colon [0, 1] \to \mathbb{R}^n$.