MATH 5703	**Topology I**	Fall 2014
Section: 001	MWF 10:45 – 11:35	Prof. Matthew Clay
	MAIN 322	

Homework 7

1. Let $M = \prod_{\mathbb{N}} [0,1]$. Show that

$$d((x_n),(x'_n)) = \sup_{n\in\mathbb{N}} \left| \frac{x_n - x'_n}{n} \right|$$

is a metric on M that induces the same topology as the product topology.

2. Suppose that S is Hausdorff, $X \subseteq S$ and there exists a continuous function $r\colon S \to X$ such that $r(x) = x$ for all $x \in X$, i.e., X is a *retract* of S. Prove that X is closed in S.

3. Suppose (M,d) is a metric space and that there exists $\epsilon > 0$ such that $\overline{B}_d(x,\epsilon)$ is compact for all $x \in M$. Prove that M is complete.

4. Suppose that S is a complete metric space, $A \subseteq S$ for which there exists a collection of open sets $\{U_n\}_{n\in\mathbb{N}}$ such that $A = \bigcap_{n\in\mathbb{N}} U_n$, i.e., A is a G_δ *set*. Show that A is a Baire space in the subspace topology.

5. Show that the set of irrational numbers $J = \mathbb{R} - \mathbb{Q}$ is a G_δ set of $\mathbb{R}$. That is, find a collection of open sets $\{U_n\}_{n\in\mathbb{N}}$ of $\mathbb{R}$ for which $J = \bigcap_{n\in\mathbb{N}} U_n$. Conclude that J is a Baire space.