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| MATH 5703 | **Topology I** | Fall 2014 |
| Section: 001 | MWF 10:45 – 11:35 | Prof. Matthew Clay |
| | MAIN 322 | |

# Homework 6

1. Prove or disprove: The product of Hausdorff spaces is Hausdorff.

2. Show that $S$ is Hausdorff if and only if the *diagonal* $\Delta = \{(x,x) \mid x \in S\} \subseteq S \times S$ is closed.

3. Show that a locally compact Hausdorff space is regular.

4. Show that if $S$ is normal and $C, D \subseteq S$ are disjoint closed subsets, then there exists open sets $U, V \subseteq S$ for which $C \subseteq U$, $D \subseteq V$ and $\overline{U} \cap \overline{V} = \emptyset$.

5. Suppose that $S$ is normal and $A \subseteq S$ is closed. Prove that there exists a continuous function $f\colon S \to [0,1]$ with $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$, if and only if there exists a collection of open sets $\{U_n\}_{n\in\mathbb{N}}$ such that $A = \bigcap_{n\in\mathbb{N}} U_n$.