MATH 5703	**Topology I**	Fall 2014
Section: 001	MWF 10:45 – 11:35	Prof. Matthew Clay
	MAIN 322	

Homework 5

1. For all $n \in \mathbb{N}$, let $S_n = \{0, 1\}$ endowed with the discrete topology and $S = \prod_{n\in\mathbb{N}} S_n$. Let

$$X = \{(x_n)_{n\in\mathbb{N}} \mid x_n = 0 \text{ for all but finitely many } n\}.$$

Prove that $\overline{X} = S$. As X is countable, this shows that S is separable.

2. Suppose that (M, δ) is a metric space and S is compact. Let M^S be the set of continuous functions from S to M and d the metric on M^S given by

$$d(f, g) = \sup_{x\in S} \delta(f(x), g(x)).$$

Show that the induced topology on M^S is the same as the compact-open topology on M^S.

3. Let $I = [0, 1] \subset \mathbb{R}$. Show that I^I, the set of continuous functions from I to I, is not compact with the metric $d(f, g) = \sup_{x\in I} |f(x) - g(x)|$.

4. Find a space that is a T_0–space but not a T_1–space.

5. Find an example of a T_1–space that contains a compact subset that is not closed. Such a space is not a T_2–space (Hausdorff).