

## Homework 4

1. Let  $C$  be a countable subset of  $\mathbb{R}^n$  where  $n \geq 2$ . Prove that  $\mathbb{R}^n - C$  is path connected.
2. Show that  $S^n$  is path connected for  $n \geq 1$ .
3. Let  $S = \{(tp, 1 - t) \in \mathbb{R}^2 \mid t \in [0, 1] \text{ and } p \in \mathbb{Q} \cap [0, 1]\}$ . Show that  $S$  is path connected but only locally connected at  $(0, 1)$ . Find a subspace  $T \subseteq \mathbb{R}^2$  that is path connected but not locally connected at any point.
4. Prove that a compact subspace of a metric space is closed and bounded.
5. Show that if  $Y$  is compact and  $C \subseteq X \times Y$  is closed, then the projection  $\pi_X(C) \subseteq X$  is closed, i.e.,  $\pi_X$  is a *closed map*. Find a counter-example to show that the statement is false if we drop the assumption that  $Y$  is compact.