

Homework 3

1. Show that if $A \subseteq X$ and $B \subseteq Y$, then $\overline{A \times B} = \overline{A} \times \overline{B}$.
2. Let X and Y be topological spaces, and assume $A \subseteq X$ and $B \subseteq Y$. Then the subspace topology on $A \times B$ inherited from $X \times Y$ is the same as the product topology on $A \times B$, where A has the subspace topology inherited from X and B has the subspace topology inherited from Y .
3. Is $X = \{(x, y) \mid x, y \geq 0 \text{ and } xy = 0\} \cup \left\{ \left(x, \frac{1}{x} \right) \mid x > 0 \right\}$ connected? Prove or disprove.
4. Let S be an infinite set and τ the *finite complement topology*, that is, $X \in \tau$ if $|S - X| < \infty$ or $X = \emptyset$. Show that S is connected.
5. Prove that if X and Y are connected, then so is $X \times Y$.