

## Homework 2

1. Suppose  $(M, d)$  and  $(N, \rho)$  are metric spaces. Show that a function  $f: M \rightarrow N$  is continuous if and only if for all  $x \in M$  and  $\epsilon > 0$ , there is a  $\delta > 0$  such that  $d(x, y) < \delta$  implies  $\rho(f(x), f(y)) < \epsilon$ .
2. Show that  $m: \mathbb{R}^2 \rightarrow \mathbb{R}$  given by  $m(x, y) = xy$  is continuous.
3. Let  $X = \left\{ \frac{1}{n} \in \mathbb{R} \mid n \in \mathbb{Z} - \{0\} \right\}$ . Show that the subspace topology is the discrete topology.
4. Let  $S$  be a topological space and let  $X \subseteq S$  have the subspace topology. Show that  $C \subseteq X$  is closed if and only if  $C = X \cap D$  where  $D \subseteq S$  is closed.
5. Suppose  $X$  is an open set of a topological space  $S$  and  $A$  is an open set of  $X$  (with the subspace topology). Show that  $A$  is open in  $S$ . Prove the same statement replacing ‘open’ with ‘closed’.