MATH 5703	**Topology I**	Fall 2014
Section: 001	MWF 10:45 – 11:35	Prof. Matthew Clay
	MAIN 322	

Homework 10

1. Let $U \subseteq \mathbb{R}^n$ be a connected open set. Show that for all $x, y \in U$ there is an embedding $p\colon [0,1] \to U$ such that $p(0) = x$, $p(1) = y$ and $p([0,1]) = |K|$ for some Euclidean complex K. Such a path is called a *polyhedral arc*.

2. Let σ^n be an n–simplex in $\mathbb{R}^n$. Show that σ^n is homeomorphic to the unit ball $\overline{B(\mathbf{0}, 1)} = \{\mathbf{x} \in \mathbb{R}^n \mid \|\mathbf{x}\| \leq 1\}$ where $\|\mathbf{x}\|$ is the standard Euclidean norm on $\mathbb{R}^n$.

For the remainder, let D^2 be the unit disk in $\mathbb{R}^2$ and let S^1 be the unit circle in $\mathbb{R}^2$.

3. Show that any homeomorphism $f\colon S^1 \to S^1$ can be extended to a homeomorphism $F\colon D^2 \to D^2$.

4. Suppose p, q, r, s are four distinct points in S^1 in the stated cyclic order and that $A, B \subset D^2$ are disjoint compact subsets such that $A \cap S^1 = \{p\}$ and $B \cap S^1 = \{r\}$. Show that q and s lie in the same component of $D^2 - (A \cup B)$.

5. Show that there is no retraction $r\colon D^2 \to S^1$. That is, show there is no continuous function $r\colon D^2 \to S^1$ such that $r(x) = x$ for all $x \in S^1$.