MATH 5703	**Topology I**	Fall 2014
Section: 001	MWF 10:45 – 11:35	Prof. Matthew Clay
	MAIN 322	

Homework 1

1. Prove that a set $X \subseteq S$ of a topological space is closed if and only if $X = \overline{X}$.

2. For a subset $X \subseteq S$ of a topological space prove the following:
   a. $\overline{S - X} = S - \text{Int}(X)$
   b. $\text{Int}(S - X) = S - \overline{X}$

3. Let S be an infinite set and suppose that τ is a topology that contains every infinite subset of S. Prove that τ is the discrete topology.

4. Consider the colllection τ of subsets of $\mathbb{R}$ consisting of the infinite intervals

$$I_a = (a, \infty) = \{x \in \mathbb{R} \mid a < x\},$$

including $I_{-\infty} = \mathbb{R}$ and $I_\infty = \emptyset$. Show that τ is topology on $\mathbb{R}$ and describe the closure of a set $X \subseteq \mathbb{R}$ in this topology.

5. Consider the function $d_\infty \colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by

$$d_\infty(\mathbf{x}, \mathbf{y}) = \max\{|x_i - y_i|\}$$

for ordered n–tuples $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$. Show that this defines a metric on $\mathbb{R}^n$ and that this metric gives the same topology as the Euclidean metric.