32. First observe that every proper subgroup of D_4 is Abelian. Now use Theorem 9.6 and Exercise 4 of Chapter 8.

56. $x(H \cap N)x^{-1} = xHx^{-1} \cap xNx^{-1} = H \cap N$. The same argument works for the intersection of any family of normal subgroups.

63. By Corollary 4 of Theorem 7.1, $x^m N = (xN)^m = N$, so $x^m \in N$.

65. Suppose that $\text{Aut}(G)$ is cyclic. Then $\text{Inn}(G)$ is also cyclic. So, by Theorem 9.4, G/Z is cyclic and from Theorem 9.3 it follows that G is Abelian. This is a contradiction.

68. The mapping $\phi : g \to xgx^{-1}$ for all g is an automorphism of G so $N = \phi(N) = xNx^{-1}$.