

8. If k divides n , then $\langle k \rangle / \langle n \rangle$ is a cyclic group of order n/k . So it is isomorphic to $Z_{n/k}$.
11. In G/H the element $(1, 1)H$ has order 4, so $G/H \approx Z_4$. In G/K the elements $(1, 1)K$, $(3, 3)K$, and $(2, 3)K$ have order 2, so $G/K \approx Z_2 \oplus Z_2$.
24. $Z_4 \oplus Z_2$.
39. Let $x \in C(H)$ and $g \in G$. We must show that $gxg^{-1} \in C(H)$. That is, for any h in H , $gxg^{-1}h = hgxg^{-1}$. Note that in the expression $(gxg^{-1})h(gxg^{-1})^{-1} = gxg^{-1}hgx^{-1}g^{-1}$ the terms x and x^{-1} cancel since $g^{-1}hg \in H$ and x commutes with every element of H . Then we have $(gxg^{-1})h(gxg^{-1})^{-1} = gxg^{-1}hgx^{-1}g^{-1} = gg^{-1}hgg^{-1} = h$. So, $gxg^{-1} \in C(H)$.
49. By Lagrange's Theorem, $|Z(G)| = 1, p, p^2$, or p^3 . By assumption, $|Z(G)| \neq 1$ or p^3 (for then G would be Abelian). So, $|Z(G)| = p$ or p^2 . However, the "G/Z" Theorem (Theorem 9.3) rules out the latter case.