

1. Closure and associativity in the product follows from the closure and associativity in each component. The identity in the product is the n -tuple with the identity in each component. The inverse of (g_1, g_2, \dots, g_n) is $(g_1^{-1}, g_2^{-1}, \dots, g_n^{-1})$.
2. Every nonidentity element in the group has order 2. Each of these generates a distinct subgroup of order 2.
3. The mapping $\phi(g) = (g, e_H)$ is an isomorphism from G to $G \oplus \{e_H\}$. To verify that ϕ is one-to-one, we note that $\phi(g) = \phi(g')$ implies $(g, e_H) = (g', e_H)$ which means that $g = g'$. The element $(g, e_H) \in G \oplus \{e_H\}$ is the image of g . Finally, $\phi((g, e_H)(g', e_H)) = \phi((gg', e_{He_H})) = \phi((gg', e_H)) = gg' = \phi((g, e_H))\phi((g', e_H))$. A similar argument shows that $\phi(h) = (e_G, h)$ is an isomorphism from H onto $\{e_G\} \oplus H$.
6. $Z_8 \oplus Z_2$ contains elements of order 8, while $Z_4 \oplus Z_4$ does not.
7. Define a mapping from $G_1 \oplus G_2$ to $G_2 \oplus G_1$ by $\phi(g_1, g_2) = (g_2, g_1)$. To verify that ϕ is one-to-one, we note that $\phi((g_1, g_2)) = \phi((g'_1, g'_2))$ implies $(g_2, g_1) = (g'_2, g'_1)$. From this we obtain that $g_1 = g'_1$ and $g_2 = g'_2$. The element (g_2, g_1) is the image on (g_1, g_2) so ϕ is onto. Finally, $\phi((g_1, g_2)(g'_1, g'_2)) = \phi((g_1g'_1, g_2g'_2)) = (g_2g'_2, g_1g'_1) = (g_2, g_1)(g'_2, g'_1) = \phi((g_1, g_2))\phi((g'_1, g'_2))$. In general, the external direct product of any number of groups is isomorphic to the external direct product of any rearrangement of those groups.