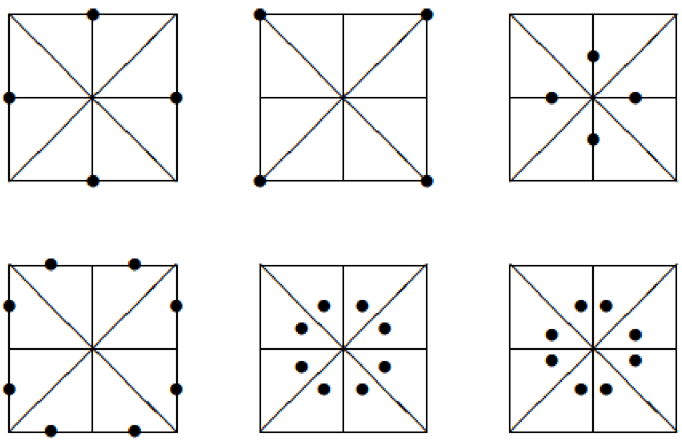
47. Consider the mapping from G to G defined by $\phi(x) = x^2$. To prove that it is one-to-one assume that $x^2 = y^2$ and let $|G| = 2k + 1$. Then $x = xe = xx^{2k+1} = x^{2k+2} = (x^2)^{k+1} = (y^2)^{k+1} = y^{2k+2} = yy^{2k+1} = ye = y$. By Exercise 12 of Chapter 5, ϕ is also onto.

49. By Corollary 3 of Lagrange's Theorem a group of order 5 is cyclic. Suppose G is a group with distinct subgroups $\langle a \rangle$ and $\langle b \rangle$ of order 5. Because 5 is prime, the identity is the only element common to the two subgroups groups. This implies that the 25 elements of the form $a^i b^j$ where $i, j \in \{0, 1, 2, 3, 4\}$ are distinct.

60.



$\{R_0, H\}$; $\{R_0, D'\}$; $\{R_0, H\}$
$\{R_0\}$; $\{R_0\}$; $\{R_0\}$.