

10. Let F and F' be distinct reflections in D_3 . Then take $H = \{R_0, F\}$ and $K = \{R_0, F'\}$.
22. Since $|H \cap K|$ must divide 12 and 35, $|H \cap K| = 1$.
43. Certainly, $a \in \text{orb}_G(a)$. Now suppose $c \in \text{orb}_G(a) \cap \text{orb}_G(b)$. Then $c = \alpha(a)$ and $c = \beta(b)$ for some α and β , and therefore $(\beta^{-1}\alpha)(a) = \beta^{-1}(\alpha(a)) = \beta^{-1}(c) = b$. So, if $x \in \text{orb}_G(b)$, then $x = \gamma(b) = \gamma(\beta^{-1}\alpha)(a) = (\gamma\beta^{-1}\alpha)(a)$. This proves $\text{orb}_G(b) \subseteq \text{orb}_G(a)$. By symmetry, $\text{orb}_G(a) \subseteq \text{orb}_G(b)$.
44. Since reflections have order 2 the subgroup must consist entirely of rotations and the subgroup of all rotations is cyclic.
45. a. $\text{stab}_G(1) = \{(1), (24)(56)\}$; $\text{orb}_G(1) = \{1, 2, 3, 4\}$
b. $\text{stab}_G(3) = \{(1), (24)(56)\}$; $\text{orb}_G(3) = \{3, 4, 1, 2\}$
c. $\text{stab}_G(5) = \{(1), (12)(34), (13)(24), (14)(23)\}$; $\text{orb}_G(5) = \{5, 6\}$