

9. Since $|a^4| = 15$, there are two cosets: $\langle a^4 \rangle$ and $a\langle a^4 \rangle$.
15. By Lagrange's Theorem the possible orders are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.
18. Note that $\phi(n) = |U(n)|$ then use Corollary 4 of Lagrange's Theorem and mimic the proof of Corollary 5 of Lagrange's Theorem.
29. The possible orders are 1, 3, 11, 33. If $|x| = 33$, then $|x^{11}| = 3$ so we may assume that there is no element of order 33. By the Corollary of Theorem 4.4, the number of elements of order 11 is a multiple of 10 so they account for 0, 10, 20, or 30 elements of the group. The identity accounts for one more. So, at most we have accounted for 31 elements. By Corollary 2 of Lagrange's Theorem, the elements unaccounted for have order 3.
33. Observe that $|G : H| = |G|/|H|$, $|G : K| = |G|/|K|$,
 $|K : H| = |K|/|H|$. So, $|G : K||K : H| = |G|/|H| = |G : H|$.