2. An automorphism of a cyclic group must carry a generator to a generator. Thus $1 \to 1$ and $1 \to -1$ are the only two choices for the image of 1. So let $\alpha : n \to n$ and $\beta : n \to -n$. Then $\text{Aut}(Z) = \{\alpha, \beta\}$.

10. Suppose α is an automorphism of G. Then $\alpha(ab) = (ab)^{-1}$ and $\alpha(ab) = \alpha(a)\alpha(b) = a^{-1}b^{-1}$. So $a^{-1}b^{-1} = (ab)^{-1} = b^{-1}a^{-1}$ for all a and b in G. Taking the inverse of both sides proves that G is Abelian.

    If G is Abelian, then for all a and b in G, we have $(ab)^{-1} = (ba)^{-1} = a^{-1}b^{-1}$. Thus $\alpha(ab) = \alpha(a)\alpha(b)$.

    That α is one-to-one and onto follows directly from the definitions.

12. $\text{Aut}(Z_2) \approx \text{Aut}(Z_1) \approx Z_1$;
    $\text{Aut}(Z_6) \approx \text{Aut}(Z_4) \approx \text{Aut}(Z_3) \approx Z_2$;
    $\text{Aut}(Z_{10}) \approx \text{Aut}(Z_5) \approx Z_4$ (see Example 4 and Theorem 6.5);
    $\text{Aut}(Z_{12}) \approx \text{Aut}(Z_8)$ (see Exercise 5 and Theorem 6.5).

16. Let ϕ be an isomorphism from G to H. For any β in $\text{Aut}(G)$ define a mapping from $\text{Aut}(G)$ to $\text{Aut}(H)$ by $\Gamma(\beta) = \phi\beta\phi^{-1}$. Then Γ is 1-1 and operation preserving. (See Theorem 0.8 and Exercise 6). To see that Γ is onto observe that for any γ in $\text{Aut}(H)$, $\Gamma(\phi^{-1}\gamma\phi) = \gamma$.

27. That α is a one-to-one follows from the fact that r^{-1} exists module n. The operation preserving condition is Exercise 9 of Chapter 0.