

1. Let  $\phi(n) = 2n$ . Then  $\phi$  is onto since the even integer  $2n$  is the image of  $n$ .  $\phi$  is one-to-one since  $2m = 2n$  implies that  $m = n$ .  $\phi(m + n) = 2(m + n) = 2m + 2n$  so  $\phi$  is operation preserving.
4.  $U(8)$  is not cyclic while  $U(10)$  is.
5. Define  $\phi$  from  $U(8)$  to  $U(12)$  by  $\phi(1) = 1$ ;  $\phi(3) = 5$ ;  $\phi(5) = 7$ ;  $\phi(7) = 11$ . To see that  $\phi$  is operation preserving we observe that  
 $\phi(1a) = \phi(a) = \phi(a) \cdot 1 = \phi(a)\phi(1)$  for all  $a$ ;  
 $\phi(3 \cdot 5) = \phi(7) = 11 = 5 \cdot 7 = \phi(3)\phi(5)$ ;  
 $\phi(3 \cdot 7) = \phi(5) = 7 = 5 \cdot 11 = \phi(3)\phi(7)$ ;  
 $\phi(5 \cdot 7) = \phi(3) = 5 = 7 \cdot 11 = \phi(5)\phi(7)$ .
6. If  $\beta$  is an isomorphism from  $G$  onto  $H$ , and  $\alpha$  is an isomorphism from  $H$  onto  $K$ , then  $\alpha\beta$  is an isomorphism from  $G$  onto  $K$ . That  $\alpha\beta$  is one-to-one and onto is done in Theorem 0.7. If  $a, b \in G$ , then  
 $(\alpha\beta)(ab) = \alpha(\beta(ab)) = \alpha(\beta(a)\beta(b)) = \alpha(\beta(a))\alpha(\beta(b)) =$   
 $(\alpha\beta)(a)(\alpha\beta)(b)$ .
20. Observe that  $\langle 2 \rangle, \langle 3 \rangle, \dots$  are distinct and each is isomorphic to  $Z$ .