

19. $\langle 1 \rangle, \langle 7 \rangle, \langle 11 \rangle, \langle 17 \rangle, \langle 19 \rangle, \langle 29 \rangle$

35.

$$\begin{array}{c} \langle p^{n-n} \rangle \\ \vdots \\ \langle p^{n-3} \rangle \\ | \\ \langle p^{n-2} \rangle \\ | \\ \langle p^{n-1} \rangle \\ | \\ \langle 0 \rangle \end{array}$$

55. $1 \cdot 4, 3 \cdot 4, 7 \cdot 4, 9 \cdot 4; x^4, (x^4)^3, (x^4)^7, (x^4)^9.$

58. The number of solutions to $x^{15} = e$ in G is 15. To see this let H be the unique subgroup of G of order 15. Then H also contains the unique subgroups of G orders 1, 3, and 5. Since every element of G that is a solution to $x^{15} = e$ has order 1, 3, 5 or 15 we know that H contains all solutions and every element of H is a solution. The same argument works when 15 is replaced by any positive integer n .

68. First note that if k is a generator then so is $-k$. Thus it suffices to show that $k \neq -k$. But $k = -k$ implies that $2k = 0$ so that $n = |k| = 1$ or 2 .