

2. For  $\langle a \rangle$ , generators are  $a$  and  $a^5$ ; for  $\langle b \rangle$ , generators are  $b, b^3, b^5,$   
and  $b^7$ ; for  $\langle c \rangle$ , generators are  $c, c^3, c^7, c^9, c^{11}, c^{13}, c^{17}, c^{19}$ .
9. Six subgroups; generators are the divisors of 20.  
Six subgroups; generators are  $a^k$ , where  $k$  is a divisor of 20.
11. By definition,  $a^{-1} \in \langle a \rangle$ . So,  $\langle a^{-1} \rangle \subseteq \langle a \rangle$ . By definition,  
 $a = (a^{-1})^{-1} \in \langle a^{-1} \rangle$ . So,  $\langle a \rangle \subseteq \langle a^{-1} \rangle$ .
14. 49. First note that the group is not infinite since an infinite cyclic  
group has infinitely many subgroups. Let  $|G| = n$ . Then 7 and  $n/7$   
are both divisors of  $n$ . If  $n/7 \neq 7$ , then  $G$  has at least 4 divisors.  
So,  $n/7 = 7$ . When 7 is replaced by  $p$ ,  $|G| = p^2$ .
43. Let  $|a| = m$ ,  $b = n$  and  $d = \gcd(m, n)$ . Then  
 $\text{lcm}(m, n) = mn/d$ ,  $|a^d| = m/d$ , and  $|b| = n$ . Then by Exercise 41,  
 $|a^d b| = \text{lcm}(m, n)$ .