51. First observe that $(a^d)^{n/d} = a^n = e$, so $|a^d|$ is at most n/d. Moreover, there is no positive integer $t < n/d$ such that $(a^d)^t = a^{dt} = e$, for otherwise $|a| \neq n$.

79. a. Suppose that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ commutes with $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. This implies that $\begin{bmatrix} a+b & a \\ c+d & c \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ a & b \end{bmatrix}$. From this we have $b = c$ and $a = b + d$. For convenience, we solve the latter equality for d in terms of a and b and get $d = a - b$. So, $C\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right) = \left\{\begin{bmatrix} a & b \\ b & a-b \end{bmatrix} \mid \text{where } a^2 - ab - b^2 \neq 0; a, b \in \mathbf{R}\right\}$.

b. Suppose that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ commutes with $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

This implies that

$$\begin{bmatrix} b & a \\ d & c \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}.$$

From this we have $b = c$ and $a = d$. So,

$$C\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} \mid \text{where } a^2 - b^2 \neq 0; a, b \in \mathbf{R}\right\}.$$

c. Suppose that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ commutes with every element of $GL(2, \mathbf{R})$. By part b we know that $c = b$ and $d = a$. So, any element in the center of $GL(2, \mathbf{R})$ has the form $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$. Also,

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}.$$

This gives

$$\begin{bmatrix} b & -a \\ a & -b \end{bmatrix} = \begin{bmatrix} -b & -a \\ a & b \end{bmatrix}.$$

So, $b = 0$. Thus,

$$Z(G) = \left\{\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \mid a \neq 0; a \in \mathbf{R}\right\}.$$

80. Let $g \in G$, $g \neq e$. If $|g| = pm$, then $|g^m| = p$.