22. $\langle 3\rangle = \{3, 3^2, 3^3, 3^4, 3^5, 3^6\} = \{3, 9, 13, 11, 5, 1\} = U(14)$. $\langle 5\rangle = \{5, 5^2, 5^3, 5^4, 5^5, 5^6\} = \{5, 11, 13, 9, 3, 1\} = U(14)$. $\langle 11\rangle = \{11, 9, 1\} \neq U(14)$.

23. Since $|U(20)| = 8$, for $U(20) = \langle k\rangle$ for some k it must the case that $|k| = 8$. But $1^1 = 1$, $3^4 = 1$, $7^4 = 1$, $9^2 = 1$, $11^2 = 1$, $13^4 = 1$, $17^4 = 1$, and $19^2 = 1$. So, the maximum order of any element is 4.

32. $H \cap K \neq \emptyset$, since $e \in H \cap K$. Now suppose that $x, y \in H \cap K$. Then, since H and K are subgroups, we know $xy^{-1} \in H$ and $xy^{-1} \in K$. That is, $xy^{-1} \in H \cap K$.

33. If $x \in Z(G)$, then $x \in C(a)$ for all a, so $x \in \bigcap_{a\in G} C(a)$. If $x \in \bigcap_{a\in G} C(a)$, then $xa = ax$ for all a in G, so $x \in Z(G)$.

35. The case that $k = 0$ is trivial. Let $x \in C(a)$. If k is positive, then by induction on k, $xa^{k+1} = xaa^k = axa^k = aa^kx = a^{k+1}x$. The case where k is negative now follows from Exercise 34. The statement "If for some integer k, x commutes a^k, then x commutes with a" is false as can be seen in the group D_4 with $x = H, a = R_{90}$ and $k = 2$.