

12. $x \rightarrow x \bmod k$ is a homomorphism with kernel $\langle k \rangle$.
21. By Theorem 10.3 we know that $|Z_{30}/\text{Ker } \phi| = 5$. So, $|\text{Ker } \phi| = 6$.
The only subgroup of Z_{30} of order 6 is $\langle 5 \rangle$.
24. a. Let $\phi(1) = k$. Then $\phi(7) = 7k \bmod 15 = 6$ so that $k = 3$ and
 $\phi(x) = 3x$.
b. $\langle 3 \rangle$.
c. $\langle 5 \rangle$.
d. $1 + \langle 5 \rangle$.
33. By property 6 of Theorem 10.1,
 $\phi^{-1}(11) = 11\text{Ker } \phi = \{11, 19, 27, 3\}$.
34. Write $U(40) = U_5(40) \times U_8(40)$ and use Exercise 9.