

7. Write $a = nq_1 + r_1$ and $b = nq_2 + r_2$, where $0 \leq r_1, r_2 < n$. We may assume that $r_1 \geq r_2$. Then $a - b = n(q_1 - q_2) + (r_1 - r_2)$, where $r_1 - r_2 \geq 0$. If $a \bmod n = b \bmod n$, then $r_1 = r_2$ and n divides $a - b$. If n divides $a - b$, then by the uniqueness of the remainder, we then have $r_1 - r_2 = 0$. Thus, $r_1 = r_2$ and therefore $a \bmod n = b \bmod n$.
11. Suppose that there is an integer n such that $ab \bmod n = 1$. Then there is an integer q such that $ab - nq = 1$. Since d divides both a and n , d also divides 1. So, $d = 1$. On the other hand, if $d = 1$, then by the corollary of Theorem 0.2, there are integers s and t such that $as + nt = 1$. Thus, modulo n , $as = 1$.
18. Observe that $8^{402} \bmod 5 = 3^{402} \bmod 5$ and $3^4 \bmod 5 = 1$. Thus, $8^{402} \bmod 5 = (3^4)^{100} 3^2 \bmod 5 = 4$.
58. $a - a = 0$; if $a - b$ is an integer k then $b - a$ is the integer $-k$; if $a - b$ is the integer n and $b - c$ is the integer m , then $a - c = (a - b) + (b - c)$ is the integer $n + m$. The set of equivalence classes is $\{[k] \mid 0 \leq k < 1, k \text{ is real}\}$. The equivalence classes can be represented by the real numbers in the interval $[0, 1)$. For any real number a , $[a] = \{a + k \mid \text{where } k \text{ ranges over all integers}\}$.
59. No. $(1, 0) \in R$ and $(0, -1) \in R$ but $(1, -1) \notin R$.