Errata for Office Hours with a Geometric Group Theorist

September 2020

Page 40. The definition of centroid is incorrect. It should defined as the point that minimizes the sum of the squares of the distances. There may be more than one point that minimizes the sum of the distances. For example, for two distinct points $a, b \in \mathbb{R}$, every point $x \in [a, b]$ minimizes $d(a, x) + d(b, x)$.

Page 47. The argument for why the inductive procedure draws the whole Farey graph is not correct, because the procedure is not invariant under the action of $\mathrm{SL}(2, \mathbb{Z})$. It is a good exercise to give a correct argument.

Page 50. The definition of a G–tiling is incomplete. We require a fourth condition: at any vertex, at most two tiles meet. Without this assumption, it is not true in Step 3 on page 53 that the path from gv to v passes through no other tiles. A simple example is given by taking the usual action of $F_2 = \langle a, b \rangle$ on its Cayley graph. If we take the tile T_0 to be the union of the edges from e to a and b, then four tiles meet at the vertex a, and so Step 2 gives too many generators.

Page 51. The definition of the T_g is incorrect. As defined, the T_g do not necessarily meet in at most one point. To make a simple counterexample, we start with the graph obtained from $\mathbb{R}$ by placing vertices at the integer points. We then attach another copy of the same graph at each vertex of the original graph. Then we consider the action of $\mathbb{Z}$ where the generator translates by 2. In this case, the intersection of two of the T_g is either empty or an infinite graph.

Here is one way to correct the definition (taking into account the fourth condition from the correction to page 50 above). First, for a vertex v of T, we define the *star* of v in T' to be the union of the edges of T' (half-edges of T) incident to v. The tree T' decomposes into the union of the stars of vertices of T. The group G acts on the set of such stars.

With this in hand, we can build up a tile T_0 containing v one star at a time. We start with the star s of v in T'. Then for each $g \in G$ we add the star gs to the tile T_g. Then we choose any star that is incident to T_0 and does not belong to any other tile, add that to T_0, and add its G–translates to the other tiles. Continuing this process inductively, we eventually obtain the desired tiling. Any two stars meet along the midpoint of an edge of T, and so at most two tiles can meet at a vertex of T'.

Page 52. The term "symmetric generating set" was not defined. This is a generating set with the property that the inverse of each generator is also a generator.

Page 55. The text says "none of these 12 matrices." The 12 should be a 6.

Page 56. Theorem 3.5 is incorrect as stated. A counterexample is the action of $\mathbb{Z}$ by translation on the graph obtained from $\mathbb{R}$ by placing vertices at the integer points. Theorem 3.5 can be corrected by adding the assumption that there are two orbits of vertices. The given proof is a correct proof of the corrected statement. The same correction is required for the statement of Theorem 3.6 on page 61. The case where there is one orbit of vertices is addressed in Project 2 on page 63.

Page 65. In Project 13 it is written that each element of $\mathrm{SL}(n, \mathbb{Z})$ is a product of (at most)

48 elementary matrices. It should be changed to saying that each element of $\mathrm{SL}(3,\mathbb{Z})$ is a product of (at most) 48 elementary matrices.

Page 84. The definition of locally extended residually finite (LERF) given in Project 8 is incorrect. The definition given in this project is equivalent to residual finiteness. A group G satisfies LERF if for every finitely generated subgroup $H \subset G$ and every $g \in G - H$, there is a finite index subgroup H' such that $H \subseteq H'$ and $g \notin H'$. This is implied by the result in discussed Project 9.

Page 84. The result discussed in Project 9 is not Marshall Hall's theorem, but a generalization of it. This result where H is the trivial group is Marshall Hall's theorem.

Page 87. In the statement of Lemma 5.2 we require $n \geq 2$.

Page 88. Exercise 3 is incorrect as stated. For example, we can take the trivial action of $\mathbb{Z}/2 \times \mathbb{Z}/2$ on the set $\{a, b\}$, we can take elements g_1 and g_2 to be any two nontrivial elements, the sets X_1 and X_2 to be both equal to $\{b\}$, and $x = a$. To correct the exercise we should replace hypothesis (2) of Lemma 5.2 with the assumption that $g_i^k(X - X_i) \subseteq X_i$.

Page 91. Figure 5.5 is incorrect. It illustrates the map $1/\bar{z}$ instead of $1/z$. If we apply complex conjugation to the salmon colored square outside the circle, and leave the rest of the picture the same, we obtain a corrected picture.

Page 98. The formula $ab - cd > 0$ should be $ad - bc > 0$.

Page 98. In the last sentence of exercise 18, the $\mathrm{PSL}(2,\mathbb{R})$ should be $\mathrm{PSL}(2,\mathbb{C})$.

Page 99. The Möbius transformation indicated in Figure 5.10 on page 101 is $z \mapsto \frac{i-z}{-1+iz}$, not $z \mapsto \frac{i-z}{i+z}$ as stated. The two transformations differ by a rotation.

Page 102. In Exercise 25, we should assume that f is not the identity.

Page 130. Part (2) of exercise 5 is incorrect. It is a good exercise to show why the given map is not a quasi-isometry.

Page 141. "A has infinite order" should be "a has infinite order."

Page 216. "$K \cong H \ltimes G$" should be "$K \cong H \rtimes G$."

Page 218. "$v \in \partial\mathcal{T}$" should be "$v \in \mathcal{T}$."

Page 227. In the definition of $\dim X \leq n$ it should be added that the refinement $\mathcal{V}$ is also an open cover of X.

Page 243. In Exercise 6 we should assume that the group G is infinite.

Page 273. "$\Omega_{\mathbb{R}^2}(\mathbb{Z})$" should be "$\Omega_{\mathbb{R}}(\mathbb{Z})$." "$\Omega_{\mathbb{R}_2}(X)$" should be "$\Omega_{\mathbb{R}^2}(X)$."

Page 276. "$\Omega_{\mathbb{R}^2}(X)$" should be "$\Omega_{\mathbb{R}^3}(X)$."

Page 287. "any element of D_3" should be "any non-trivial element of D_3."

Page 299. The map $G(P_3) \to B_4$ sending v_i to σ_{i+1} is not a homomorphism as neither σ_1 nor σ_3 commute with σ_2. The graph P_3 should be replaced with the graph complement Γ of P_3. That is, Γ has three vertices v_0, v_1 and v_2 and one edge connecting v_0 to v_2. The map $G(\Gamma) \to B_4$ defined by $v_i \mapsto \sigma_{i+1}$ is a homomorphism and is not injective as σ_1 and σ_2 do not generate a free subgroup.

Page 300. In the presentation for PB_3, "A_{23}" should be "A_{13}."

Page 329. After the displayed matrix for gt, the phrase "the coefficients of P are exactly the same in gt as in t" should be "the coefficients of P are exactly the same in gt as in g."

Page 366. In the sentence "It follows that a compact orientable surface without boundary is determined up to homeomorphism by any two of the three numbers χ, g, and b," the phrase "without boundary" should be removed.

We are grateful to Yiftach Barnea, Hyeran Cho, Ethan Dlugie, Alex Kastner, Jean-Francois LaFont, Ian Leary, Lily Li, Kasra Rafi, Michah Sageev, Dmytro Savchuk, Matan Seidel, Culler Shaffer, Christopher-Lloyd Simon, Jiajie Tao, and Christian Urech for corrections.