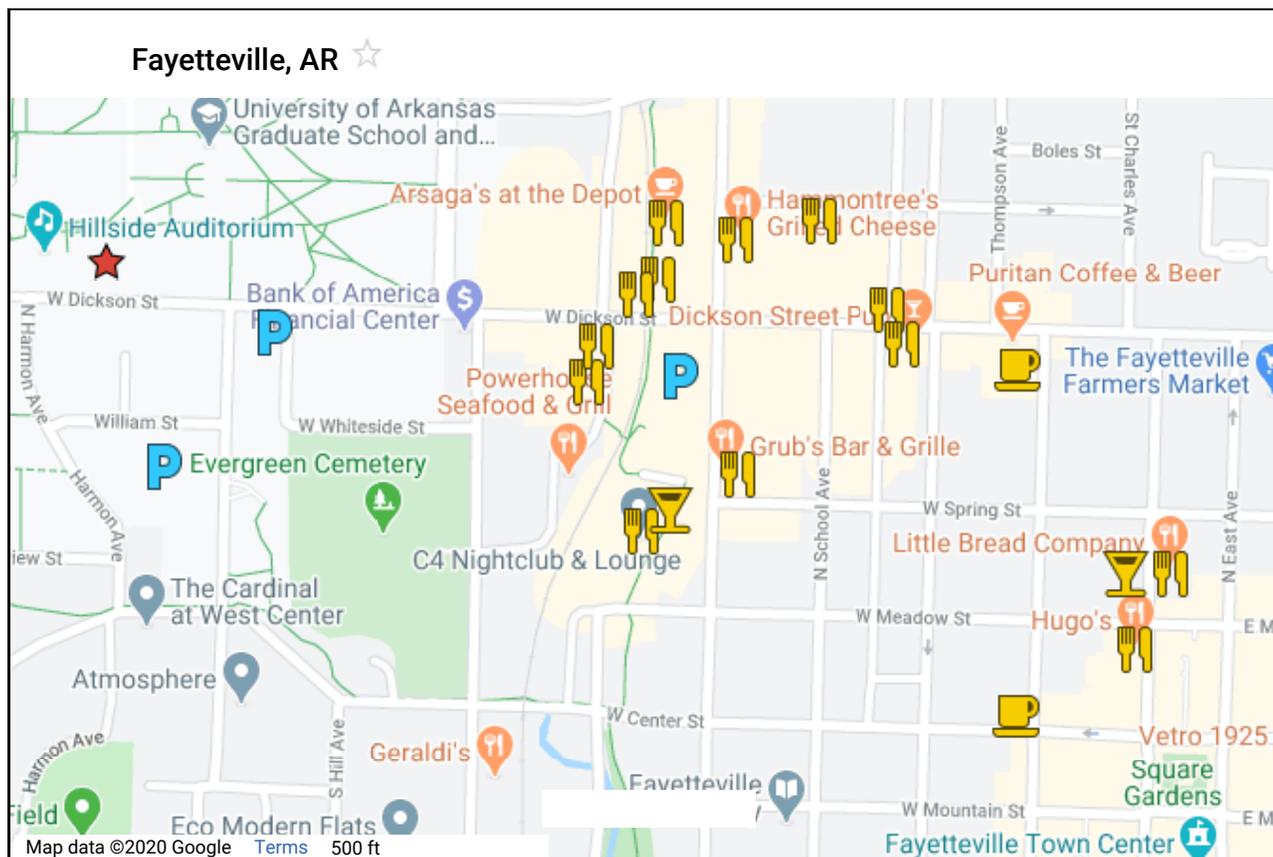


# Redbud Topology Conference

University of Arkansas, Fayetteville, Arkansas: March 6 – 8, 2020



## Restaurants near to campus:

Arsaga's	548 W Dickson St.	local organic (\$\$)
Bordinos	310 W Dickson St.	italian (\$\$\$)
Emelia's Kitchen	540 W Dickson St.	mediterrian (\$\$ - dinner only)
Hammontree's	326 N West Ave.	fancy grilled cheese (\$\$)
Grub's Bar and Grille	220 N West Ave.	burgers (\$)
Jimmy John's	518 W Dickson St.	sandwiches (\$)
Marley's	609 W Dickson St.	pizza (\$\$)
Rolando's	509 W Spring St.	latino (\$\$)
Saffron	401 W Watson St., Ste 207	indian (\$\$)
Spoon	603 W Dickson St.	korean (\$\$)

**Banquet:** Saturday at the Garden Room (215 W Dickson St.). There will be a cash bar starting at 6:00PM; food will be served at 6:30PM.

**Schedule:** The talks are on the University of Arkansas campus in the room as indicated. Abstracts follow.

## Friday, March 6 (Student Workshop SCEN 408)

Time	Speaker	Talk
1:00 – 1:50	Susan Hermiller	<a href="#">Computability and computational complexity</a>
2:10 – 3:00	Anastasiia Tsvietkova	<a href="#">Complexity theory and 3–manifolds</a>
3:20 – 4:10	Nathan Dunfield	<a href="#">Algorithms for studying 3–manifolds: theory and practice</a>
4:10 – 4:45	<b>break</b>	
4:45 – 5:30		<a href="#">Lightning Talks</a>

**Nathan Dunfield**, University of Illinois, Urbana-Champaign

**Title:** Algorithms for studying 3–manifolds: theory and practice

**Abstract:** I will sketch some foundational tools for building 3–manifold algorithms, namely normal surface theory and hyperbolic geometry. I will explain how Haken used normal surfaces to give an algorithm for determining the genus of a knot and the work of Agol–Hass–Thurston and Lackenby which shows this question is in the intersection of NP and coNP. I will then give a few hints on how Kuperberg showed the Geometrization Theorem allows one to solve the homeomorphism problem for compact 3–manifolds. I'll conclude with a quick demonstration of the programs Regina and SnapPy to convince you that, regardless of their theoretical complexity, many natural questions can be rapidly answered in practice.

**Susan Hermiller**, University of Nebraska

**Title:** Computability and computational complexity

**Abstract:** In computer science, Turing machines are used to define the concept of computability. Finite state automata, on the other hand, model computers with a finite amount of memory. In this talk I will discuss FSA's, Turing machines, and a hierarchy of other computational models between them.

**Anastasiia Tsvietkova**, Rutgers University

**Title:** Complexity theory and 3–manifolds

**Abstract:** Topology and geometry of 3–manifolds has many problems with an easy formulation, but a hard solution. Despite our intuitive feeling that these problems are "hard", lower or upper bounds on algorithmic complexity are known only for some of them. We will first review the basics of algorithmic complexity theory for problems with discrete input. Then we will list some known results and open problems concerning algorithmic complexity and 3–manifolds.

## Saturday, March 7 (SCEN 408)

Time	Speaker	Talk
8:00 – 8:30	coffee	
8:30 – 9:20	David Futer	<a href="#">Special covers of alternating links</a>
9:30 – 10:20	Nathan Dunfield	<a href="#">Counting incompressible surfaces in 3-manifolds</a>
10:20 – 10:50	coffee	
10:50 – 11:40	Susan Hermiller	<a href="#">Fundamental groups of 3-manifolds and finite state automata</a>
11:50 – 12:40	William Worden	<a href="#">Small knots of large Heegaard genus</a>
12:40 – 2:00	lunch	
2:00 – 2:50	Ian Agol	<a href="#">RFRS for congruence subgroups</a>
3:05 – 3:55	Jennifer Schultens	<a href="#">Surface complexes</a>
3:55 – 4:30	break	
4:30 – 5:00		<a href="#">Lightning Talks</a>

## Sunday, March 8 (SCEN 408)

Time	Speaker	Talk
8:00 – 8:30	coffee	
8:30 – 9:20	Eric Samperton	<a href="#">How helpful is hyperbolic geometry?</a>
9:30 – 10:20	Anastasiia Tsvietkova	<a href="#">Some NP-hard problems naturally arising in knot theory</a>
10:20 – 10:50	coffee	
10:50 – 11:40	Eric Sedgwick	<a href="#">Embeddability in <math>\mathbb{R}^3</math> is NP-hard</a>
11:50 – 12:40	Greg Kuperberg	<a href="#">Algorithmic homeomorphism of 3-manifolds is elementary recursive</a>

**Ian Agol**, University of California, Berkeley

**Title:** RFRS for congruence subgroups

**Abstract:** Addressing a question of Baker and Reid, we show that infinitely many Bianchi groups admit congruence towers satisfying the Residually Finite Rational Solvable (RFRS) condition. In particular the associated Bianchi orbifolds have congruence covers fibering over the circle. We also show the RFRS condition for congruence towers of some arithmetic 4-dimensional lattices, implying virtual algebraic fibering by a result of Kielak. This is joint work with Matthew Stover.

**Nathan Dunfield**, University of Illinois, Urbana-Champaign

**Title:** Counting incompressible surfaces in 3-manifolds

**Abstract:** Counting embedded curves on a hyperbolic surface as a function of their length has been much studied by Mirzakhani and others. I will discuss analogous questions about counting incompressible surfaces in a hyperbolic 3-manifold, with the key difference that now the surfaces themselves have more intrinsic topology. As there are only finitely many incompressible surfaces of bounded Euler characteristic up to isotopy in a hyperbolic 3-manifold, it makes sense to ask how the number of isotopy classes grows as a function of the Euler characteristic. Using Haken's normal surface theory and facts about branched surfaces, we can characterize not just the rate of growth but show it is (essentially) a quasi-polynomial. Moreover, our method allows for explicit computations in reasonably complicated examples. This is joint work with Stavros Garoufalidis and Hyam Rubinstein.

**David Futer**, Temple University

**Title:** Special covers of alternating links

**Abstract:** The "virtual conjectures" in low-dimensional topology, stated by Thurston in 1982, postulated that every hyperbolic 3-manifold has finite covers that are Haken and fibered, with large Betti numbers. These conjectures were resolved in 2012 by Agol and Wise, using the machine of special cube complexes. Since that time, many mathematicians have asked for a quantitative statement: just how big does a cover need to be in order to ensure one of these desired properties?

We begin to give a quantitative answer to this question, in the setting of alternating links in  $S^3$ . If a prime, alternating link  $K$  has a diagram with  $n$  crossings, we prove that the complement of  $K$  has a special cover of degree less than  $n!$ . Corollaries of this result include a quantification of residual finiteness, explicit control of the growth of Betti numbers in covers, an explicit bound on the rank of a  $\mathbb{Z}$ -module on which the link group acts faithfully. This is joint work with Edgar Bering.

**Susan Hermiller**, University of Nebraska

**Title:** Fundamental groups of 3-manifolds and finite state automata

**Abstract:** In this talk I will discuss several ways to solve the word problem for groups by finite automata, including automatic and autostackable structures, along with geometric and topological views of these properties. We apply these algorithms to fundamental groups of 3-manifolds. Based on joint projects with M. Brittenham and T. Susse, and with D. Holt, S. Rees, and T. Susse.

**Greg Kuperberg**, University of California, Davis

**Title:** Algorithmic homeomorphism of 3-manifolds is elementary recursive

**Abstract:** I'll say a few words about the folklore theorem, which was essentially known to Thurston, that geometrization implies an algorithm to distinguish closed 3-manifolds. Then I will discuss the refinement that this computational task is elementary recursive, meaning that the work is bounded by a bounded tower of exponentials. The least standard part of the proof is for the hyperbolic case. In this case I will describe a strategy based on straightening and self-refining a non-geometric triangulation of a hyperbolic 3-manifold to produce a geometric triangulation.

**Eric Samperton**, University of Illinois, Urbana-Champaign

**Title:** How helpful is hyperbolic geometry?

**Abstract:** Greg Kuperberg and I previously found examples of NP-complete decision problems for homology 3–spheres, and also for complements of knots in  $S^3$ . I'll report on work in progress I've done to show that these problems are still hard when restricted to hyperbolic homology 3–spheres, or complements of hyperbolic knots.

The technical problem is to build high distance Heegaard splittings locally, where these splittings must satisfy the same restrictive conditions relevant to complexity theory that Greg and I exploited before. The techniques also work to give analogous results for certain quantum invariants of hyperbolic 3–manifolds, and give life to the phrase "hyperbolic quantum computing."

**Jennifer Schultens**, University of California, Davis

**Title:** Surface complexes

**Abstract:** Curve complexes of surfaces have been successfully used to study automorphism groups and 3–manifolds. Building on their success, we consider higher dimensional analogues. One such analogue derives from the Kakimizu complex of a knot. While the Kakimizu complex of a knot provides one avenue to discuss surfaces in a 3–manifold, other complexes should also be considered. We discuss both the Kakimizu complex and another surface complex in the context of Seifert fibered spaces.

**Eric Sedgwick**, DePaul University

**Title:** Embeddability in  $\mathbb{R}^3$  is NP-hard

**Abstract:** We prove that the problem of deciding whether a 2–or 3–dimensional simplicial complex embeds into  $\mathbb{R}^3$  is NP-hard. This stands in contrast with the lower dimensional cases which can be solved in linear time, and a variety of computational problems in  $\mathbb{R}^3$  like unknot or 3–sphere recognition which are in  $\text{NP} \cap \text{co-NP}$  (assuming the generalized Riemann hypothesis). Our reduction encodes a satisfiability instance into the embeddability problem of a 3–manifold with boundary tori, and relies extensively on techniques from low-dimensional topology, most importantly Dehn fillings on link complements. This is joint work with Arnaud de Mesmay (CNRS, GIPSA-Lab, France), Yo'av Rieck (University of Arkansas, USA) and Martin Tancer (Charles University, Czech Republic).

**Anastasiia Tsvietkova**, Rutgers University

**Title:** Some NP-hard problems naturally arising in knot theory

**Abstract:** Many problems that lie at the heart of classical knot theory can be formulated as decision problems. We consider such problems related to unlinking and splitting by crossing changes, to Reidemeister moves, and to detecting alternating links and sublinks. We show that many of these problems are NP-hard. This is joint work with Dale Koenig.

**William Worden**, Rice University

**Title:** Small knots of large Heegaard genus

**Abstract:** Building off ideas developed by Agol, we construct a family of hyperbolic knots  $K_n$  whose complements contain no closed incompressible surfaces (i.e., they are small) and have Heegaard genus exactly  $n$ . These are the first known examples of small knots having large Heegaard genus. In the first part of the talk we will describe a beautiful construction due to Agol for building hyperbolic 3–manifolds that decompose into a union of regular ideal octahedra. Using this technology, we will then show how to build the knots  $K_n$ , and outline the proof that they have the desired properties.

## Lightning Talks:

### Friday:

Alex Casella, Florida State University  
Coordinates for real projective structures using ideal triangulations

Joshua Howie, University of California, Davis  
An algorithm to decide if a link is alternating

Rachel Lehman, University of Arkansas  
A structure theorem for bad 3-orbifolds

Pravakar Paul, University of Iowa  
An examples of Khovanov–Floer theory

Puttipong Pongtanapaisa, University of Iowa  
Weakly reducible and destabilized bridge spheres for knots

Brandon Bavier, Michigan State University  
Weakly generalized alternating knots and knot invariants

### Saturday:

Jean Pierre Mutanguha, University of Arkansas  
Ergodic automorphisms of free groups

Rylee Lyman, Tufts University  
Recognizing pseudo-Anosov braids in  $\text{Out}(W_n)$

Lorenzo Ruffoni, Florida State University  
A rank formula for generalized Bestvina-Brady subgroups of RAAGs

Derrick Wigglesworth, University of Arkansas  
Algorithmic detection of geometric free group automorphisms

Ryan Spitler, McMaster University  
Profinite rigidity of manifolds and groups